NRL Report 7650

Some Fracture Mechanics Relationships for Thin Sheet Materials

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December 21, 1973

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CONTENTS

ABSTRACT

The fracture resistance parameter K_c has been determined for a number of aluminum, titanium, and steel alloy sheet materials over a thickness range of 0.032 to 0.25 in. (0.8 to 6.25 mm). Several interesting K_c relationships have been developed from these investigations.

The K_c parameter is found to depend inversely upon the material yield stress. Further, a relationship can be established between K_c and fracture appearance. Analysis of the data has also disclosed that the amount of crack extension, i.e., final crack length $2a_c$ appears to be influenced by the initial crack length. A straight-line curve in logarithmic coordinates relates the ratio of initial to final crack length, a_0/a_c to $1/\beta_c$; this is justified by statistical analyses.

The development of these relationships can be of real assistance in the design of a standard initial screening test for K_c.

PROBLEM STATUS

This is a final report on one phase of a continuing NRL problem.

AUTHORIZATION

NRL Problem M01-24
Project RR 022-01-46-5431

Manuscript submitted August 27, 1973.

SYMBOLS

$2a$, $2a_0$, $2a_c$	Crack or notch length in sheet; subscripts 0 and c refer respectively to initial and critical values
r_y	Plastic zone size
r_{xy}	Correlation coefficient
A	Cross-sectional area of specimen, BW
B	Specimen thickness
B_F	Amount of flat fracture
B_{SL}	Amount of slant (or shear) fracture
CCT	Center-cracked tension specimen
E	Young's modulus
F	Statistical distribution
K, K_c, K_{Ic}	Fracture resistance parameter; subscripts c and Ic refer to critical values under conditions of plane stress and plane strain, respectively
P	Load
W	Specimen width
$\mathcal{G}$, $\mathcal{G}_c$, $\mathcal{G}_{Ic}$	Crack extension force; subscripts c and Ic refer to critical values under conditions of plane stress and plane strain, respectively
β_c	$\frac{1}{B}\left(\frac{K_c}{\sigma_{ys}}\right)^2$
σ_G	Gross or nominal stress, P/A
σ_{ys}	Yield stress

SOME FRACTURE MECHANICS RELATIONSHIPS FOR THIN SHEET MATERIALS

INTRODUCTION

The parallel demands of today's structural engineers for high strength and light weight in thin sheet materials imposes the obligation of insuring against catastrophic fracture by considering the fracture resistant properties of such materials. Fracture resistance can be defined as the ability of a material to withstand the deleterious effects of cracks, flaws, or notches while under load. One measurement of fracture resistance in thin sheet materials is the plane-stress fracture mechanics parameter K_c. Unlike other mechanical properties, however, the accurate determination of K_c is a function of K_c itself, since specimen size depends upon the ratio of K_c to yield stress, K_c/σ_{ys}. Other geometrical factors involved are crack length-to-width ratio $2a/W$ and material thickness B. These interconnected dependencies preclude the development of a single standard test specimen unless one is chosen so large as to be economically indefensible.

Since research conducted over the past three years at NRL has established guidelines to assist in the estimation of allowable specimen width and crack length-to-width ratio, it is appropriate to consider the relationships between K_c, yield strength, and fracture appearance. Further, since crack growth occurs under load in the K_c specimen, some analyses of crack growth characteristics seem advisable.

EXPERIMENTAL PARAMETERS

Materials

The materials studied include eight aluminum, four titanium, and five steel alloys in sheet form with thicknesses ranging from 0.032 to 0.25 in. (0.8 to 6.25 mm). Full details of the mechanical and fracture resistance properties are available (1-13). Relevant data are presented in Tables 1-4. In all specimens the fracture path was parallel to the rolling direction of the sheet (*TL*).

Test Procedure

The center cracked tensile (CCT) specimen was utilized for the determination of K_c. Crack opening is measured by a strain-gage-instrumented displacement probe positioned in a circular hole in the center of the initial slit. This displacement measurement (COD) is referred to a normalized experimental calibration curve which relates the amount of crack opening to the instantaneous crack length of the specimen. Both load and COD are simultaneously graphed by an x-y recorder until failure occurs. These techniques have been previously discussed (1,5). Values of slant fracture percent were calculated from measurements made by a micrometer slide comparator at three positions ahead of the critical crack.

Table 1
Fracture Resistance Data for Aluminum Alloys

Alloy	B (in.)	W (in.)	$2a_0$ (in.)	$2a_c$ (in.)	σ_{ys} (ksi)	K_c (ksi $\sqrt{\text{in.}}$)	Percent Slant	β_c‡
7178-T6	0.040	12	2.130	2.40	79	49	53	9.7
	0.063	12	2.124	2.40	78	50	50	6.4
	0.091	12	2.138	2.40	78	44	50	3.6
	0.127	12	2.140	2.54	77	45	44	2.7
7075-T6	0.032	12	2.130	2.54	71	52	100	16.4
	0.063	12	2.220	2.23	76	60	—	—
	0.063	12	2.20	2.52	—	58	—	—
	0.063	12	4.14	4.50	—	63	—	—
	0.063	12	4.23	4.56	—	62	61	10.1
	0.093	12	2.094	2.46	74	65	65	8.2
	0.102	12	2.154	2.56	75	62	61	6.8
	0.126	12	2.116	3.04	76	55	56	4.1
	0.198	12	2.098	3.26	76	56	46	2.8
	0.250	12	2.130	2.70	74	46	45	1.6
7079-T6	0.037	12	2.138	3.66	68	62	84	21.9
	0.063	12	2.138	3.42	71	71	79	15.7
	0.101	12	2.128	3.90	62	98	78	24.9
	0.140	12	2.144	3.12	72	68	48	6.5
	0.250	12	2.086	3.34	66	81	41	6.0
2014-T6	0.040	12	2.156	3.18	62	75	100	37.4
	0.063	12	2.120	3.06	61	72	100	22.0
	0.091	12	2.129	3.10	57	75	91	19.1
	0.125	12	2.100	3.80	60	92	100	19.1
	0.250	12	2.118	3.20	59	66	89	4.9
2219-T87	0.032	12	2.150	3.81	52	86	100	84.4
	0.062	12	2.155	3.44	54	77	100	32.9
	0.091	12	2.137	3.76	51	92	100	35.6
	0.125	12	2.139	4.59	52	88	100	22.8
	0.250	12	2.120	3.60	54	76	100	8.0
7178-T6†	0.063	12	2.132	2.576	78	36	42	3.5
	0.091	12	2.190	2.784	76	45	32	3.9
	0.125	12	2.145	3.28	78	48	34	2.9
	0.25	12	2.16	2.16	78	36	23	0.84
	0.30	12	2.135	2.88	79	36	17	0.71
7075-T6†	0.063	12	2.132	2.64	72	70	44	14.8
	0.092	12	2.150	3.136	76	77	50	11.5
	0.124	12	2.132	2.96	74	61	38	5.4
	0.25	12	2.168	2.168	76	42	25	1.2
	0.30	12	2.132	2.85	74	46	21	1.3

†Rolled down from 0.30-in. sheet.

‡$\beta_c = \frac{1}{B}(K_c/\sigma_{ys})^2$.

Table 2
Fracture Resistance Data for Titanium Alloys

Alloy	B (in.)	W (in.)	$2a_0$ (in.)	$2a_c$ (in.)	σ_{ys} (ksi)	K_c (ksi$\sqrt{\text{in.}}$)	Percent Slant	β_c†
6A1-4V	0.032	12	2.138	2.82	151	54	100	4.00
	0.062	12	2.132	2.70	158	64	100	2.86
	0.090	12	2.132	2.40	146	77	100	2.66
4A1-3Mo-1V	0.042	12	2.144	2.144	162	50	68	2.30
	0.058	12	2.234	2.234	153	54	59	2.30
	0.087	12	2.290	2.290	152	62	54	1.90
	0.124	12	2.132	2.132	160	35	46	.38
16V-2.5A1	0.041	12	2.128	2.52	182	52	100	2.00
	0.059	12	2.137	2.137	176	46	52	1.16
	0.118	12	2.138	2.64	182	44	53	.51
13V-11A-3A1	0.040	12	2.152	2.152	198	34	23	.74
	0.063	12	2.113	2.113	207	38	26	.56
	0.091	12	2.116	2.116	201	30	12	.24
	0.125	12					11	.09

†$\beta_c = \frac{1}{B}(K_c/\sigma_{ys})^2$

Data Analysis

Fracture resistance K_c is calculated according to equation (Ref. 14)

$$K_c = \sigma_G\sqrt{a_c}\ f\ (2a/W), \tag{1}$$

where

$$f\,(2a/W) = 1.77\left[(1 - 0.1\left(\frac{2a}{W}\right) + \left(\frac{2a}{W}\right)^2\right]. \tag{2}$$

Values of σ_G and $2a_c$ are determined from the load and COD measurements mentioned above.

Regression lines, correlation coefficients and t and F statistical calculations were computed from conventional statistical definitions and equations (15,16).

DEGRADATION OF K_c WITH INCREASED YIELD STRENGTH

The fact that materials show a degradation in resistance to fracture K_{Ic} with increasing yield stress has long been recognized for thick plate material. As can be seen in Fig. 1, the same inverse relationship also exists for thin sheet alloys. Upper and lower limit lines define the highest and lowest K_c values measured with respect to yield strength for the alloy systems of aluminum, titanium, and steel. (Thicknesses from 0.032 to 0.25 in. are included.) The ratio lines K_c/σ_{ys} are accompanied by estimates of the minimum width requirement for the CCT specimen. Normalizing the data shown in Fig. 1 by the yield

Table 3
Fracture Resistance Data for Steels

Steel	B (in.)	W (in.)	$2a_0$[+] (in.)	$2a_c$[†] (in.)	σ_{ys} (ksi)	K_c (ksi $\sqrt{\text{in.}}$)	Percent Slant	$\beta_{c\ ave}$[‡§]
4130	0.032	12	2.18	4.00	168	151	100	21.80
	0.032	12	4.51	6.84	168	146	100	
	0.050	12	2.21	4.56	172	172	100	20.10
	0.050	12	4.17	6.72	172	174	100	
	0.063	12	2.26	3.78	170	172	100	18.20
	0.063	12	4.21	6.54	170	191	100	
	0.087	12	2.20	3.26	183	124	78	4.85
	0.087	12	4.18	6.07	183	144	84	4.85
	0.125	12	2.21	3.54	176	148	91	6.80
	0.125	12	4.18	5.86	176	158	92	
4130	0.032	12	2.13	4.98	185	154	100	22.6
	0.032	12	4.15	6.66	185	161	100	
	0.051	12	2.15	3.16	185	146	100	13.60
	0.051	12	4.10	6.60	185	158	100	
	0.063	12	2.16	3.72	178	129	83	9.40
	0.063	12	4.25	6.12	178	146	—	
	0.087	12	2.10	3.26	200	123	—	4.50
	0.087	12	4.08	6.46	200	127	71	
	0.125	12	2.10	3.72	191	155	—	4.60
	0.125	12	4.10	4.92	191	138	77	
	0.250	12	2.20	4.22	185	163	—	2.30
	0.250	12	4.16	5.40	185	121	20	
D6A	0.098	12	2.10	2.10	228	54	21	0.66
	0.098	12	4.09	4.09	228	62	20	
	0.190	12	2.12	2.12	220	48	10	0.23
	0.190	12	4.12	4.12	220	45	9	
	0.25	12	2.16	2.16	230	68	14	0.45
	0.25	12	4.14	4.14	230	86	16	
RSM-250	0.063	12	2.162	2.68	244	180	100	10.0
	0.063	12	4.154	5.94	244	207	100	
	0.090	12	2.226	2.94	246	204	100	7.6
	0.090	12	4.120	4.75	246	204	100	
	0.140	12	2.126	3.04	248	194	100	4.2
	0.140	12	4.154	5.04	248	186	100	

[+]Average values of a_0/a_c at each thickness were used for statistics.
[†]Average value of K_c at each thickness was used for β calculation.
[§]$\beta_c = \frac{1}{B}(K_c/\sigma_{ys})^2$.

Table 4
Fracture Resistance Data for Aluminum 7475-T61

B (in.)	W (in.)	$2a_0$ (in.)	$2a_c$ (in.)	σ_{ys} (ksi)	K_c (ksi$\sqrt{\text{in.}}$)	β_c‡	$2a_c$§ (in.)	K_c# (ksi$\sqrt{\text{in.}}$)	Percent Difference
0.063	12	2.13	3.96	68	123†	52.8	3.62	116	−5.7
0.063	12	3.13	5.04	68	124†	52.8	5.32	128	+3.2
0.107	12	2.12	3.07	72	103	19.1	3.26	106	+2.9
0.107	12	3.10	4.14	72	105	19.9	4.78	117	+11.4
0.190	12	2.16	2.94	72	102	9.0	3.06	105	+2.9
0.190	12	3.09	4.14	72	87	7.6	4.30	90	+3.4
0.25	12	2.13	3.26	72	96	7.0	2.94	91	−5.2
0.25	12	3.07	4.24	72	102	8.0	4.28	104	+2.0
0.063	20	3.61	5.56	68	130	60.6	5.20	120	−7.7
0.063	20	5.10	6.60	68	111	42.4	8.48	132	+18.9
0.109	20	3.62	4.70	72	107	20.4	5.60	118	+10.2
0.109	20	5.10	6.90	72	108	20.8	7.88	118	+9.2
0.190	20	3.60	4.20	72	80	6.4	4.92	88	+10.0
0.190	20	5.13	5.60	72	76	5.8	6.96	88	+15.8
0.25	20	3.62	4.50	72	88	5.9	4.92	93	+5.6
0.25	20	5.11	6.64	72	99	7.4	7.10	104	+5.0

†σ net $> \sigma_{ys}$

‡$\beta_c = \frac{1}{B}(K_c/\sigma_{ys})^2$

§Values calculated from Eq. (1).

#Values derived from calculated value of $2a_c$.

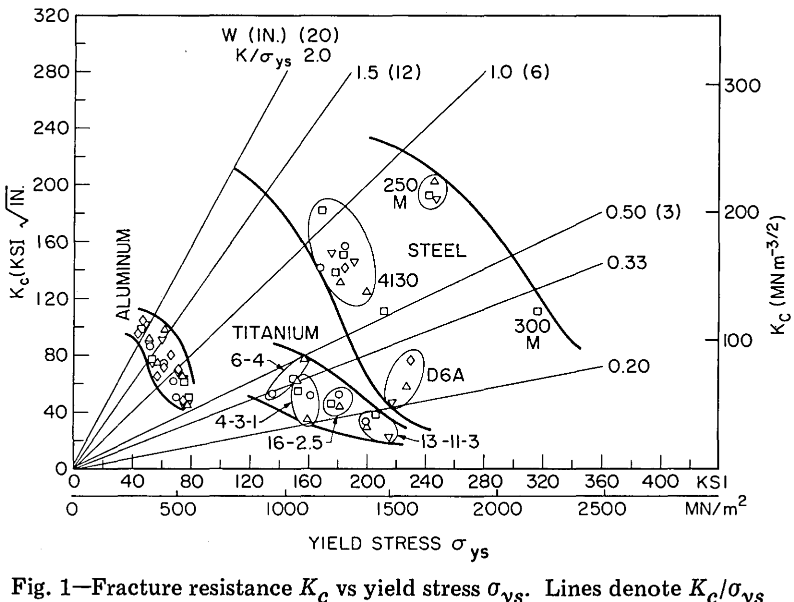


Fig. 1—Fracture resistance K_c vs yield stress σ_{ys}. Lines denote K_c/σ_{ys} ratio. An estimate of minimum specimen width W accompanies each ratio value

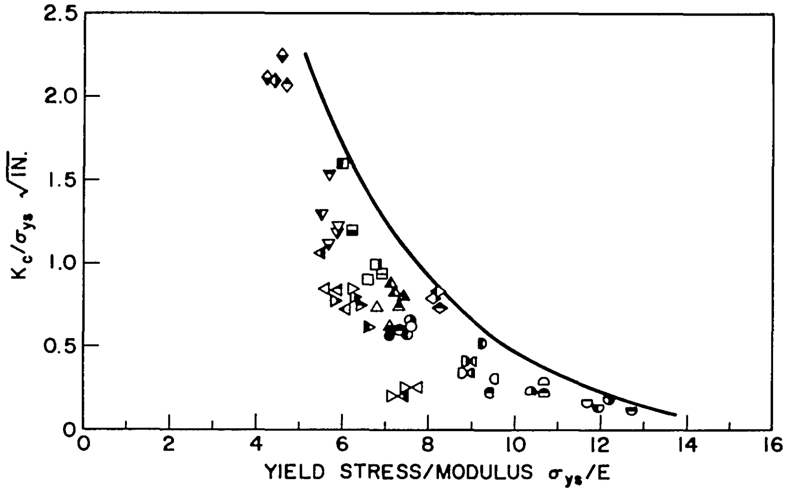


Fig. 2—Fracture resistance vs yield stress normalized as K_c/σ_{ys} vs σ_{ys}/E

strength and elastic modulus gives the curve shown in Fig. 2. A similar curve could be plotted using yield stress and alloy density as normalizing factors.

These diagrams indicate the range of values observed. From them it is possible, knowing yield stress and modulus, to estimate an upper limit for the K_c of an unknown material so that an appropriate test specimen width can be selected. It can be noted that a "standard" width of 12 in. is sufficient for values of $K/\sigma_{ys} \leqslant 1.5$.

DEPENDENCE OF K_c ON FRACTURE APPEARANCE

Although attempts to describe the relationship of K_c and sheet thickness in terms of the two models proposed (17,18) have so far not proven satisfactory (6,7,10-12) it was noted and reported (6) that for titanium alloys there appeared to be some relationship between K_c and percent slant fracture within a wide scatterband. Reexamination of the data indicated that K_c values from test specimens containing both blunt and sharp notches had been incorporated in these initial plots. Accordingly, the data from three alloy systems were replotted using the sharp notch data only, with the results seen in Figs. 3a, 3b, 3c, and 3d.

However, inasmuch as slant or shear fracture indicates the involvement of an energy process, it seemed more reasonable to utilize the relationship $K_c{}^2 = \mathcal{G}_c E$ and replot these data using the $\mathcal{G}_c$ term. Figures 4a, 4b, 4c, and 4d disclose the unexpected result that the linear relationship between $\mathcal{G}_c$ and percent slant appears to go through the origin. The schematic diagram of K vs crack growth seen in Fig. 5 indicates an increasing K value with no crack growth until an initiation value K_{Ic} is reached and thereafter, increasing values of K terminating in final K_c after some amount of crack growth. Since the K_{Ic} (or $\mathcal{G}_{Ic}$) value is normally associated with a flat fracture appearance, that is, zero percent shear, it would have been reasonable to expect positive values of $\mathcal{G}$ for each alloy at zero percent slant. However, since this was not the case, the slopes of the curves drawn were utilized to calculate $\mathcal{G}_c$ from percent slant according to the following straight-line equations:

$$\text{Steel } \mathcal{G}_c = 740\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} \tag{3}$$

$$\text{Aluminum } \mathcal{G}_c = 550\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} \tag{4}$$

$$\text{Titanium } \mathcal{G}_c = 240\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B}\,. \tag{5}$$

$\mathcal{G}_c$ was then converted to K_c values through the expression $K = \sqrt{\mathcal{G}E}$. The relationship between the calculated and measured values plotted in Fig. 6 was analyzed statistically. Although the regression curve K (calc.) = 0.84 K (meas.) + 5.24 does not quite go through the origin, the correlation coefficient is high; r_{xy} = 0.92.

Considering that the so-called $\mathcal{G}_c$ (or K_c value) is the total value, it was decided to analyze the data by subtracting $\mathcal{G}_{Ic}$ values such that

$$\mathcal{G}_c^* = \mathcal{G}_c - \mathcal{G}_{Ic}\left(1 - \frac{B_{SL}}{B}\right) \tag{6}$$

$$= \mathcal{G}_c - \mathcal{G}_{Ic}\left(\frac{B_F}{B}\right) \tag{7}$$

The $\mathcal{G}_{Ic}$ values were estimated from the yield stress values of the materials utilizing relationships found in several NRL reports (19,20).

Figures 7a, 7b, and 7c show plots of these values against percent slant. Again, the curves appear to go through the origin and the same ordering of alloy systems persists, although the slopes are slightly altered to give the following set of straight-line equations:

$$\text{Steel } \mathcal{G}_c^* = 730\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} \tag{8}$$

$$\text{Aluminum } \mathcal{G}_c^* = 570\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} \tag{9}$$

$$\text{Titanium } \mathcal{G}_c^* = 195\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} \tag{10}$$

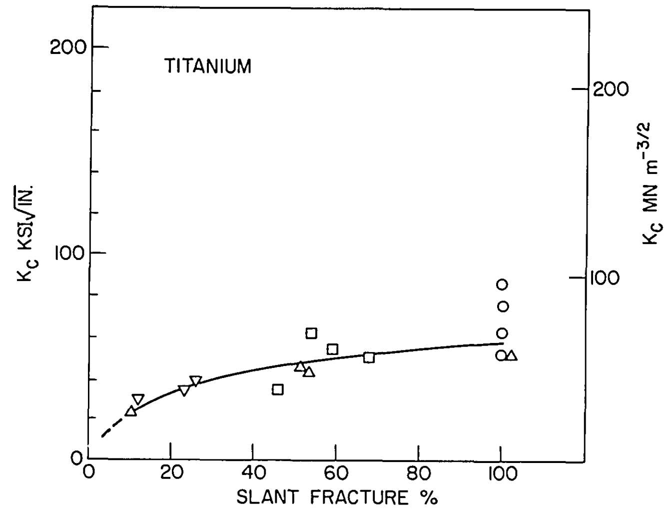
Regression analyses performed on these data for comparison with the drawn curves gave the following relationships:

$$\text{Steel } \mathcal{G}_c^* = 729\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} + 46\left(\frac{\text{in.-lb}}{\text{in.}^2}\right) \tag{11}$$

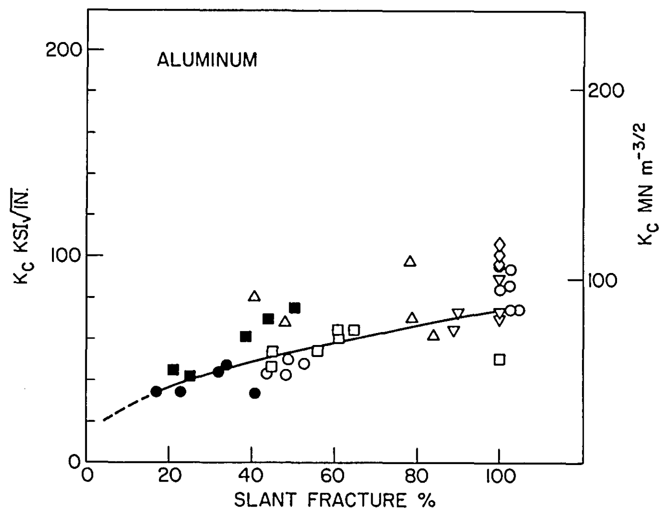
$$\text{Aluminum } \mathcal{G}_c^* = 582\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} + 24\left(\frac{\text{in.-lb}}{\text{in.}^2}\right) \tag{12}$$

$$\text{Titanium } \mathcal{G}_c^* = 192\left(\frac{\text{in.-lb}}{\text{in.}^2}\right)\frac{B_{SL}}{B} + 6.6\left(\frac{\text{in.-lb}}{\text{in.}^2}\right). \tag{13}$$

The correlation coeffocient r_{xy} was also obtained for each alloy system; these values are

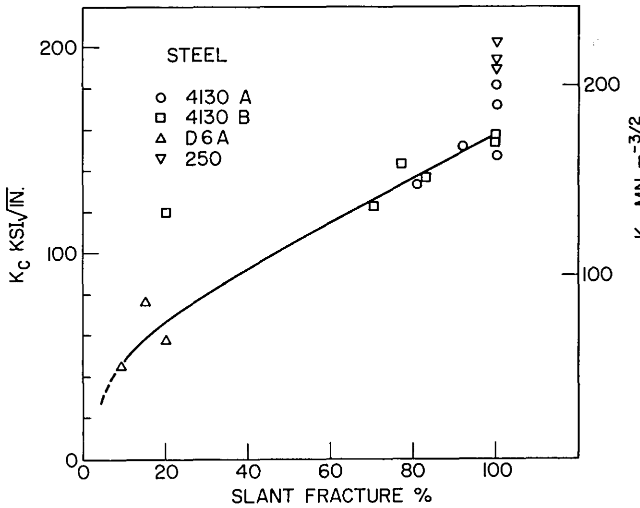


a. Aluminum

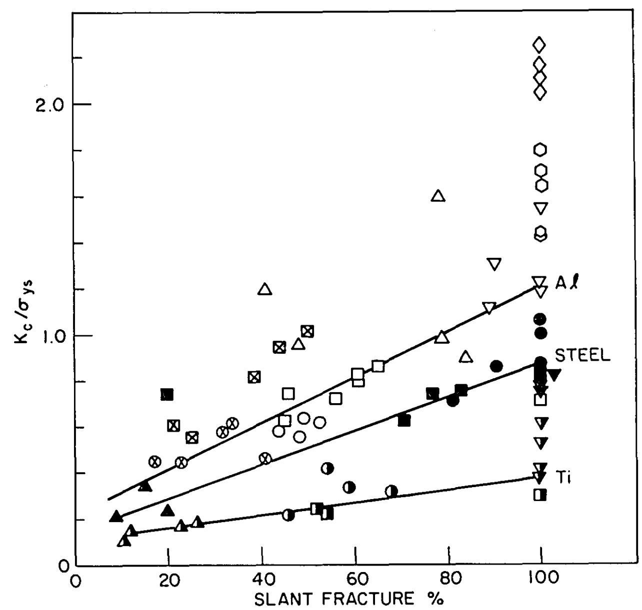


b. Titanium

Fig. 3—Fracture resistance K_c vs percent slant fracture

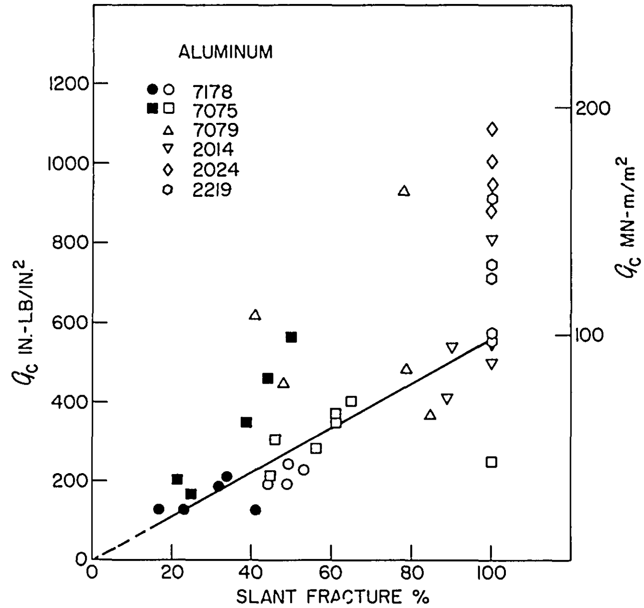


c. Steel

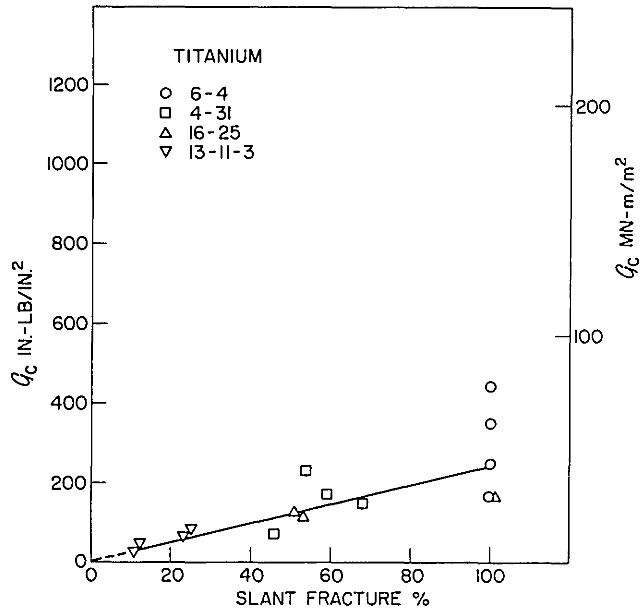


d. Normalized for all alloys as K_c/σ_{ys}

Fig. 3 (*Continued*)—Fracture resistance K_c vs percent slant fracture

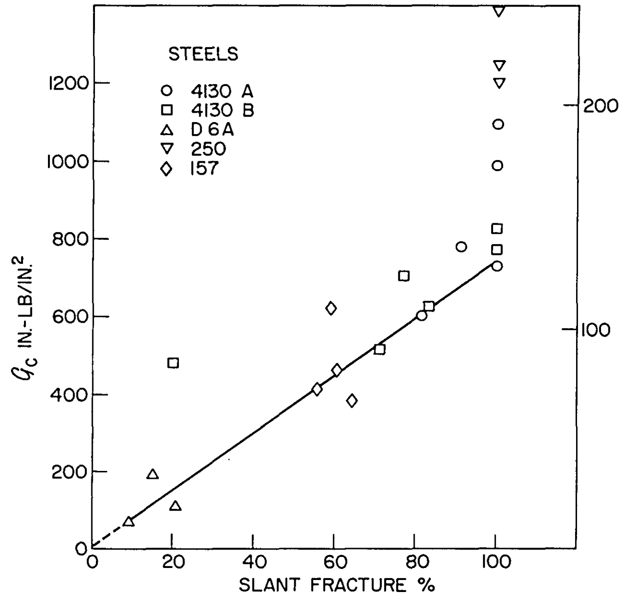


a. Aluminum

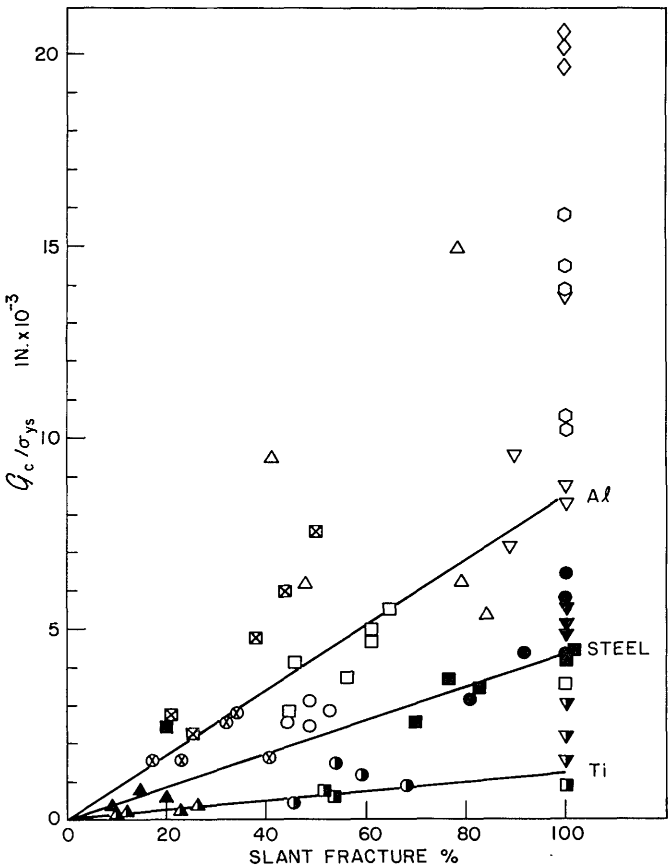


b. Titanium

Fig. 4—Crack extension force $\mathcal{G}_c$ vs percent slant fracture



c. Steel



d. Normalized as $\mathcal{G}_c/\sigma_{ys}$

Fig. 4 (*Continued*)—Crack extension force $\mathcal{G}_c$ vs percent slant fracture

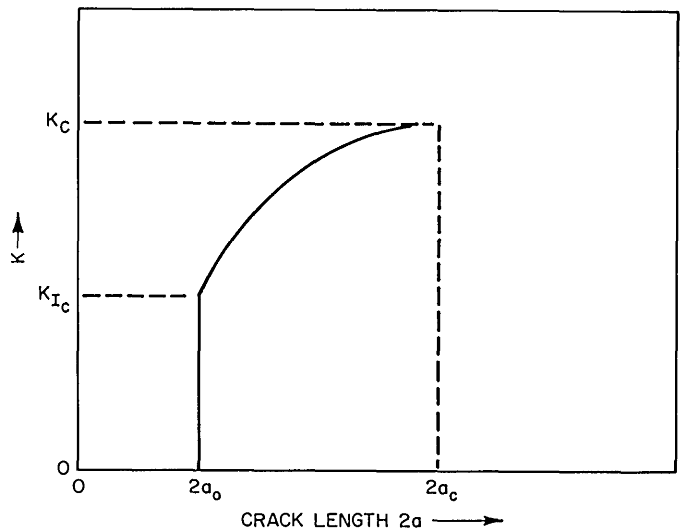


Fig. 5—Relationship of K_{Ic} and K_c to specimen crack length

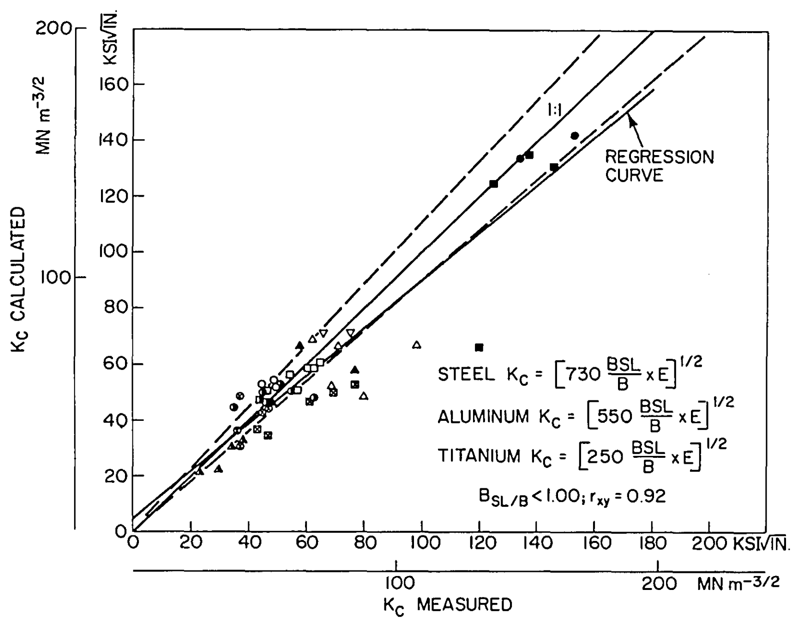
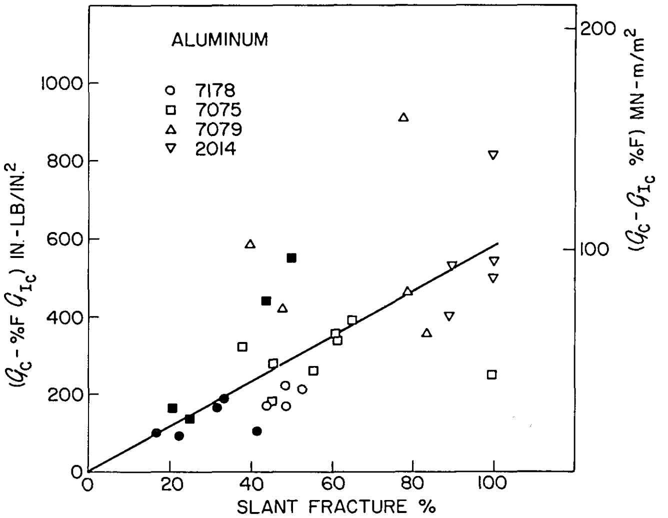
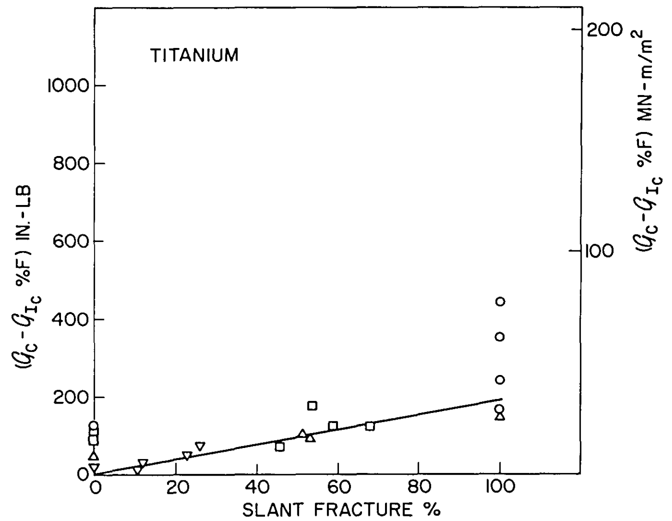


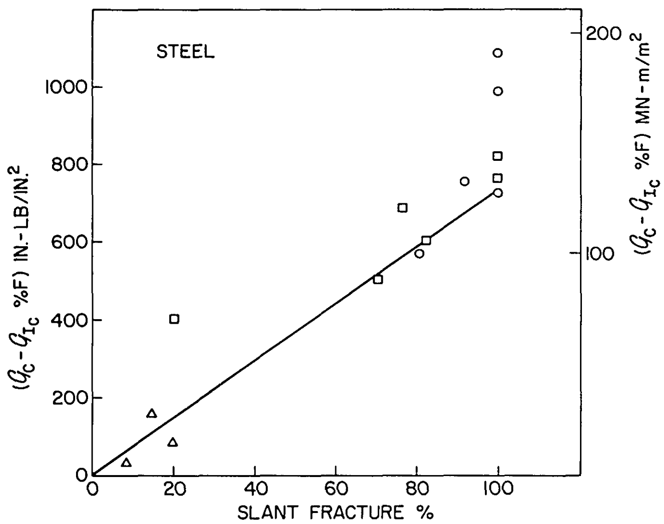
Fig. 6—K_c calculated vs K_c measured for all alloys. Equations (3), (4), and (5) used for calculations.



a. Aluminum

b. Titanium





c. Steel

Fig. 7—$\mathcal{G}_c^*$ [= $\mathcal{G}_c$ - $\mathcal{G}_{Ic}(B_F/B)$] vs percent slant fracture

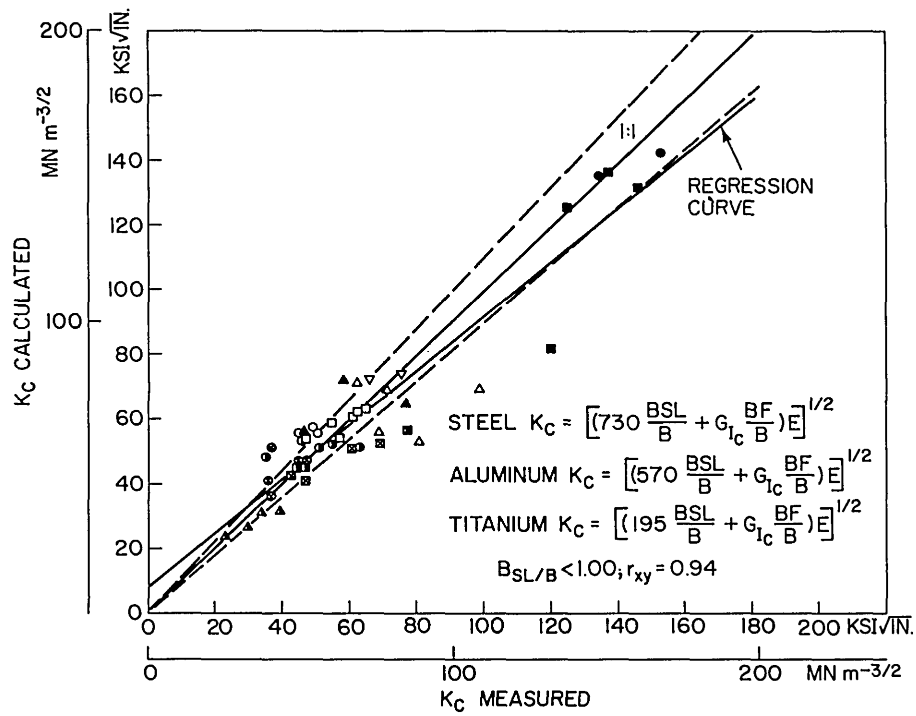


Fig. 8—Calculated K_c vs measured K_c for all alloys. Equations (8), (9), and (10) used for calculation.

$$\text{Steel } r_{xy} = 0.92$$
$$\text{Aluminum } r_{xy} = 0.63$$
$$\text{Titanium } r_{xy} = 0.76.$$

In view of the estimates involved, these values indicate definite correlations.

$\mathcal{G}_c$ values (here $\mathcal{G}_c = \mathcal{G}_c^* + \mathcal{G}_{Ic}(B_F/B)$) were again calculated and converted to K_c values. For simplicity, Eqs. (8), (9), and (10) were employed for this purpose. The calculated and measured values are compared in Fig. 8. Statistical analyses of these data give a regression curve

$$K_c\ (\text{calc.}) = 0.84\ K_c\ (\text{meas.}) + 8.2, \tag{14}$$

with a correlation coefficient of $r_{xy} = 0.94$.

The two models, I and II, earlier proposed consider the slant fracture contribution as a volume-sensitive mechanism. Model II suggests the relationship

$$\mathcal{G}_c = \mathcal{G}_c^{**} \frac{B_{SL_0}^2}{B} + \mathcal{G}_{Ic} \frac{B_F}{B}, \tag{15}$$

where B_{SL_0} = critical (constant) shear-lip thickness. Since a constant value for the amount of shear lip was rarely attained in the thickness series of alloys reported here, data for $(\mathcal{G}_c - \mathcal{G}_{Ic})B_F/B$ against B_{SL}^2/B were computed and regression analyses determined. Values of the correlation coefficient for the three alloy systems are

$$\text{Steel } r_{xy} = 0.90$$

$$\text{Aluminum } r_{xy} = 0.38$$

$$\text{Titanium } r_{xy} = 0.53.$$

From the relationships here developed, δ_c is seen constant for each given alloy system when $B_{SL} < B$. The ordering of the systems precludes normalization by either Young's modulus or density. Since the alloy systems also represent different crystallographic systems, it is suggested that these may influence the ordering, perhaps through the rolling textures developed. Clarification of this point might be assisted by investigations of fracture resistance in specimens with the fracture path transverse to the rolling direction (LT).

RELATIONSHIP BETWEEN INITIAL AND FINAL CRACK LENGTH

When a sheet specimen containing a notch is loaded in tension, the stress must reach a certain value before a crack will form at the notch tips. Once such a crack has initiated, it will grow under a rising load until the instability value is reached. Since computation of K_c, the fracture resistance value, requires knowledge of both stress and crack length at failure, crack length must be measured during the course of the test. A method for estimating the amount of crack extension from the initial notch length would be of practical value since it would eliminate the necessity for crack extension measurement. The question to be resolved is whether or not the final crack length $2a_c$ is influenced by the initial crack length $2a_0$. Certain authors (21,22) have taken the position that no influence should exist, whereas others (23-26) indicate that an influence does exist. The results of this investigation suggest a form for a relationship between $2a_c$ and $2a_0$.

When final crack length is plotted against initial crack length, a straight-line relationship can be observed. This is illustrated in Figs. 9a and b for aluminum alloy 7075-T6; other materials indicate a similar trend. Attempts to correlate this slope value with other parameters showed a relationship with the dimensionless value of

$$\beta_c = \left[\frac{1}{B}\left(\frac{K_c}{\sigma_{ys}}\right)^2\right].$$

Since the effect of thickness upon the value of K_c has been studied for a series of alloys, this information was analyzed.

Plotting the data on linear coordinate paper (Fig. 10) suggests the possibility of a linear-logarithmic relationship. Such a plot is seen in Fig. 11, together with the calculated regression curve and confidence limits.

For purposes of arithmetical expedience, the data were analyzed in the form

$$\ln 10\left(\frac{a_0}{a_c}\right) = A + B \ln 10\beta_c. \tag{16}$$

The regression curve calculated for 59 datum points is

$$\ln 10\left(\frac{a_0}{a_c}\right) = 2.4367 - 0.1142 \ln 10\beta_c, \tag{17}$$

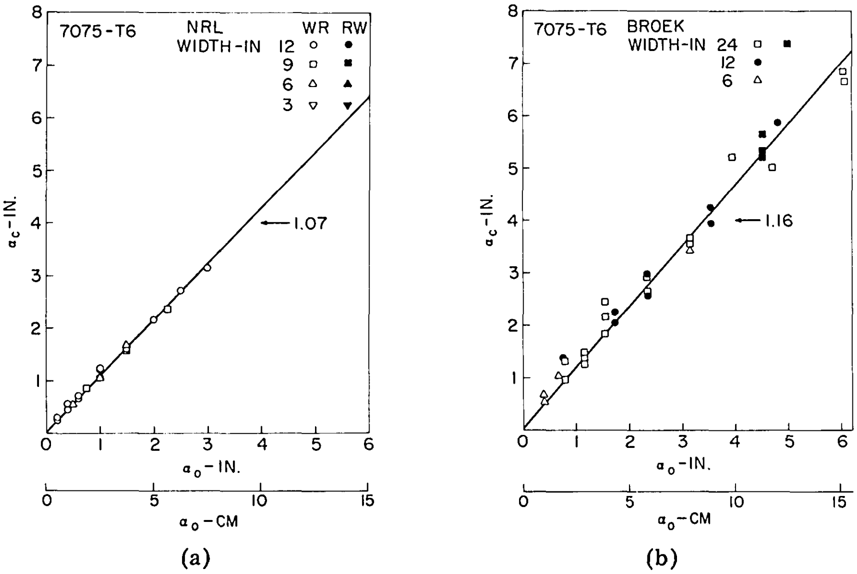


Fig. 9—Half critical crack length a_c vs half initial crack length a_0

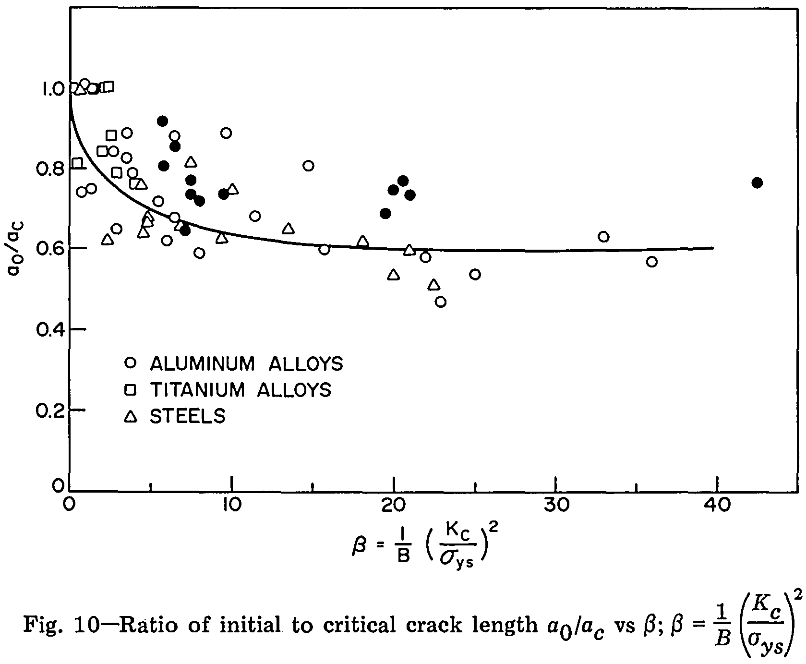


Fig. 10—Ratio of initial to critical crack length a_0/a_c vs β; $\beta = \dfrac{1}{B}\left(\dfrac{K_c}{\sigma_{ys}}\right)^2$.

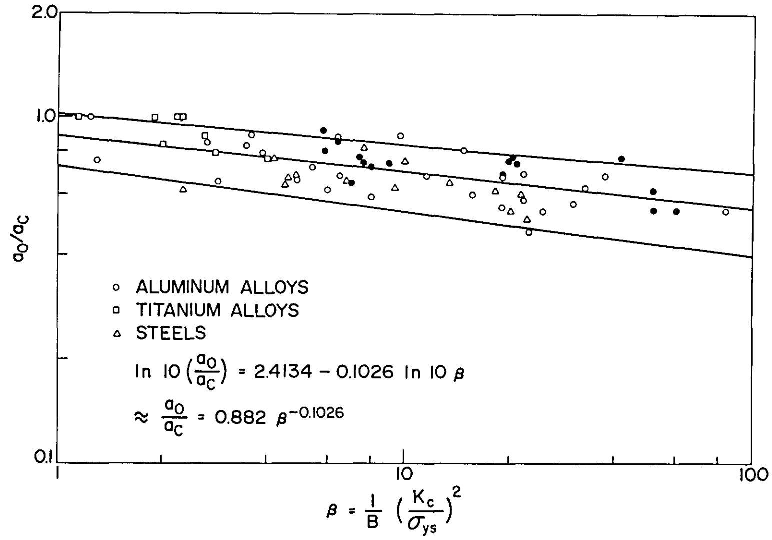


Fig. 11—Log a_0/a_c vs log β. Regression curve flanked by 95% confidence limit lines; closed symbols are for aluminum 7475-T61.

with a standard deviation for $y = 0.21$ and a correlation coefficient $r = 0.75$. A t-statistic computed from the correlation coefficient equals -8.56, which is outside the 99% confidence interval of $t = \pm 2.648$; this indicates that a relationship does exist between a_0/a_c and β.

Since the data comprised values obtained from testing aluminum, titanium and steel alloys, these data were also analyzed separately and intercompared. An analysis of variance ratios (F-statistic) indicates the following.

1. One regression line may be used for all three alloy systems;

$$F = 3.16 < F\ 99\% = 3.65.$$

2. The slopes of the three systems are equal;

$$F = 1.118 < F\ 99\% = 4.98.$$

3. Regression of the means is linear;

$$F = 6.53 < F\ 99\% = 7.08.$$

4. Slopes within groups are equal to the slope among groups;

$$F = 4.44 < F\ 99\% = 7.08.$$

Finally, another set of aluminum alloy data was added, bringing the number of datum points to 75. The regression curve computed is

$$\ln 10\left(\frac{a_0}{a_c}\right) = 2.4134 - 0.1026 \ln 10\beta_c, \qquad (18)$$

with a standard deviation for $y = 0.20$ and a correlation coefficient $r = 0.71$. This equation was used to compute a_0/a_c for the last set of aluminum data (alloy 7475-T61) and K_c values from the estimates of $2a_c$. These estimates are included in Table 4. The fact that these are slightly high is rationalized by the fact that certain values of $a_0/a_c = 1.00$ were included in the analysis. When no crack growth occurs with the sharp Elox notch tip, a fatigued crack might well have indicated slight growth and thus altered the constants in the regression curve. Nonetheless, it is believed that this analysis indicates the dependence of crack extension upon the initial crack and the feasibility of using such a relationship for estimation.

Because the final crack length $2a_c$ employed for these analyses is the "effective" crack length determined from a COD calibration and includes a plastic zone factor, one might envision this relationship between initial and final crack length as indicating that the plastic zone size is influenced by some constraint factor in the specimen, since $\beta = 2\pi r_y/B$.

SPECIMEN SCREENING

The relationship between K_c and σ_{ys} shown in Fig. 1 indicates the minimum specimen width required for various ratios of K_c/σ_{ys} (1). However, it is suggested that a width of 12 in. is adequate for the majority of high-strength materials.

The steps of a possible screening procedure are outlined below.

1. Estimate K_c from the relationship between K_c and σ_{ys} (Figs. 1 or 2).

2. Calculate $\beta_c = \frac{1}{B}\left(\frac{K_c}{\sigma_{ys}}\right)^2$.

3. Determine a_0/a_c from Fig. 11 and Eq. (18).

4. Select an initial crack length $2a_0$ such that $2a_c$ will be less than $2a/W = 0.5$ (a generally acceptable crack length-to-width ratio is 0.3).

5. Perform the test; that is, load the specimen to fracture, recording the maximum load.

6. Compute K_c from the following equation:

$$K_c = \sigma_{G_{\max}} \sqrt{a_0\left(\frac{a_c}{a_0}\right)}\; f\; 2a/W. \tag{19}$$

CONCLUSIONS

1. Degradation of K_c with increased yield stress is demonstrated by an inverse relationship.

2. Fracture resistance is directly related to the percent of slant fracture when $B_{SL} < B$, such $\mathcal{G}_c^* = (\mathcal{G}_c - \mathcal{G}_{Ic})\, B_F/B = A\,(B_{SL}/B)$.

3. Values of the constant A for the different alloy systems suggest dependence upon the sheet rolling textures.

4. Final crack length at instability is influenced by the initial crack length.

5. The final crack length-initial crack length relationship is statistically acceptable in the form $\ln 10\,(a_0/a_c) = A - B \ln 10\beta_c$ and can be transformed to $a_0/a_c = A/\beta_c^B$.

6. The relationship between a_0/a_c and β_c suggests that the plastic zone size varies with crack length.

7. Estimation of the final crack length permits a simplification of K_c determination for an initial screening test.

ACKNOWLEDGMENT

The authors gratefully acknowledge the assistance of Dr. L. Wiener, Mathematics and Information Sciences Division, NRL, in providing consultation for and corroboration of the statistical analysis, and that of the Office of Naval Research for their financial support of these studies.

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Research Laboratory Washington, D.C. 20375	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

SOME FRACTURE MECHANICS RELATIONSHIPS FOR THIN SHEET MATERIALS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Final report on one phase of a continuing NRL Problem.

5. AUTHOR(S) *(First name, middle initial, last name)*

A.M. Sullivan and J. Stoop

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 21, 1973	25	26

8a. CONTRACT OR GRANT NO. NRL Problem M01-24 b. PROJECT NO. RR 022-01-46-5431 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) NRL Report 7650 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Department of the Navy Office of Naval Research Arlington, Va. 22217

13. ABSTRACT

The fracture resistance parameter K_c has been determined for a number of aluminum, titanium, and steel alloy sheet materials over a thickness range of 0.032 to 0.25 in. (0.8 to 6.25 mm). Several interesting K_c relationships have been developed from these investigations.

The K_c parameter is found to depend inversely upon the material yield stress. Further, a relationship can be established between K_c and fracture appearance. Analysis of the data has also disclosed that the amount of crack extension, i.e., final crack length $2a_c$ appears to be influenced by the initial crack length. A straight-line curve in logarithmic coordinates relates the ratio of initial to final crack length, a_0/a_c to $1/\beta_c$; this is justified by statistical analyses.

The development of these relationships can be of real assistance in the design of a standard initial screening test for K_c.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Plane-stress fracture resistance						
Fracture mechanics parameter K_c						
Yield stress						
Fracture appearance						
Slant or shear fracture						
Crack propagation						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification